

# Understanding the structure of rational numbers: A CSCL environment to facilitate conceptual change

Xenia Vamvakoussi<sup>1</sup>, Georgios Kargiotakis<sup>2</sup>, Vassilios Kollias<sup>3</sup>,  
Nektarios G. Mamalougos<sup>1</sup>, Stella Vosniadou<sup>1</sup>

<sup>1</sup> Cognitive Science and Educational Psychology Laboratory, University of Athens,

<sup>2</sup> University of Peloponnese,

<sup>3</sup> University of Thessaly,

[xenva@phs.uoa.gr](mailto:xenva@phs.uoa.gr), [g.kargiotakis@ellinoagliko.edu.gr](mailto:g.kargiotakis@ellinoagliko.edu.gr), [vkollias@pre.uth.gr](mailto:vkollias@pre.uth.gr),  
[nektar@phys.uoa.gr](mailto:nektar@phys.uoa.gr), [svosniad@cc.uoa.gr](mailto:svosniad@cc.uoa.gr)

## ABSTRACT

*Prior knowledge of natural numbers often stands in the way of children's understanding rational numbers and their properties. In particular, the idea of discreteness is a fundamental presupposition of children's initial theories about numbers. Understanding of the dense structure of rational numbers requires radical reorganization of children's prior knowledge, namely conceptual change. In this study we implemented a CSCL environment to support 9<sup>th</sup> grade student's understanding of the dense structure of rational numbers. We investigated whether particular characteristics of the learning environment would facilitate conceptual change. We report the results of this study.*

**KEY WORDS:** *conceptual change, rational numbers, density, CSCL environment*

## INTRODUCTION

Already at a very young age, children have formed what has been characterized as “principled understanding” of natural numbers (Gelman, 2000), which is based on the act of counting. A basic characteristic of this initial “theory” of numbers is that numbers are discrete in nature. In terms of the conceptual change theoretical framework proposed by Vosniadou (1994, 2001), discreteness is a *fundamental presupposition* of children's initial “theory” of numbers. In previous work (Vamvakoussi and Vosniadou, in press), we have argued that understanding the dense structure of rational numbers requires conceptual change. In this study we implemented a CSCL environment to facilitate 9<sup>th</sup> grade students' understanding of density. This environment will give students the opportunity:

- to express their ideas about the structure of the set of rational numbers
- to externalise their visual representation of the structure of rational numbers
- to see their fellow students' ideas and realise that these can be quite different than their own
- to argue with fellow students relative to the structure of the set of natural numbers

In line with teaching guidelines coming from the conceptual change literature (Vosniadou 2002, 2001b), we expect that these factors will facilitate conceptual change.

## METHOD

The intervention took place at a public high school, in the area of Athens. The participating students were 30 9<sup>th</sup> graders, all belonging to the same class. The experimental group consisted of 16 students, working in groups of two at the school computer lab. Each group used one PC account in the software Synergeia ([http://www.euro-cscl.org/site/itcole/public\\_deliverables\\_html](http://www.euro-cscl.org/site/itcole/public_deliverables_html)). SYNERGEIA is a data base having both synchronous and asynchronous tools where students can upload and share files and participate in structured discussions. It has been developed and tested through the European project ITCOLE (<http://www.euro-cscl.org/site/itcole>). Students were familiar with computers, but they had no experience with collaborative learning or with CSCL. The control group consisted of 14 students, working in groups of two, in their classroom. The participating teachers (one mathematics and one computer science teacher) were experienced and pedagogically informed. Both groups participated in four 45-minute sessions, during which they dealt with the same tasks. The Synergeia group participated in a 20-minute introductory session, during which students got acquainted with the use of Synergeia.

Before the intervention, students took a pre-test related to their understanding of the structure of the set of rational numbers. We used a paper-and-pencil questionnaire, developed in previous study (Vamvakoussi and Vosniadou, in press). The same questionnaire was used as post-test. Decisions on how to group students were made by the teachers, according to the results of the pretest and the social dynamics in the classroom. According to our design, teachers set the initial questions of the inquiry and participated in the discussion, but did not provide any information on the mathematical content and did not evaluate students' ideas. The tasks designed by the teachers are presented on Table 1.

Session	Task	Description
1 <sup>st</sup>	“What do you know about the number line (of real numbers)? Describe as best as you can. Read and comment upon the answers of your fellow students.”	Express prior knowledge about the number line
2 <sup>nd</sup>	“We often use the term “the set of real numbers”. Suppose someone tries to understand what we mean by that. Could you draw a picture to help him/her understand?”	Construct a model for the set of real numbers
3 <sup>rd</sup>	“We have been talking about two different representations of real numbers: A “formal” one, which we usually use at school, and a second one, which was proposed in our discussion and you seem to find adequate. Could you find a solid reason why we should prefer one over the other?”	Compare two given models
4 <sup>th</sup>	“Imagine that you can become as small as a point of the number line. Then you could see the other points really close. Suppose that you are on the point that stands for the number 2.3. Can you define what point is the one closest to you? Describe in words or by drawing a picture.”	Construct a model of the number line

**Table 1:** The initial tasks posed by teachers

The experimental group students presented their work in the environment of Synergeia. The control group students presented their work in the classroom. Students who worked in the environment of Synergeia had the opportunity to reflect on their own and their fellow students' ideas. We consider this to be the added value of Synergeia to the learning environment we

implemented. After the post-test, each student was asked to comment (in written) on whether he/she believed he/she had answered both pre- and post-tests in the same way and whether he/she believed he/she had learnt something new during the intervention. Along with the pre- and post-tests, students' and teacher's notes in SYNERGEIA, audio taping of students' discussions in their groups and final comments of the students were used as sources of data.

## RESULTS

According to the results of the pre-test, there was no significant difference in the performance of the experimental and the control group before the intervention. After the intervention, there was a significant improvement in the performance of the Synergeia group (Wilcoxon signed ranks test,  $p < 0.05$ ). On the contrary, there was no significant improvement in the performance of the control group.

Although the time spent in the environment of Synergeia was quite restricted and there was not much opportunity for students to be engaged in long discussions, they seemed to be interested in reading their fellow students' notes and comment on them. While few comments were adequately explained and supported by valid arguments, they made clear that students were aware of the differences between their own and their fellow students' answers. On the contrary, students in the control group usually missed such differences. Moreover, the experimental group students were able to make deeper evaluation of their learning progress. For example, "I found out that numbers are more complicated than I thought" (Synergeia group student); "I did not learn anything new, but I revised my opinions" (Synergeia group student); "It was just a recapitulation of what we have learnt at school" (typical answer of control group students).

## DISCUSSION

The biggest challenge we had to face when implementing the CSCL environment was time: We did not manage to arrange sessions more than 45 minutes each (one school hour). Several sessions we had scheduled were rescheduled or called off. Finally, apart from being shorter and less than we would like, sessions were usually scheduled far apart from one another. Still, the improvement of the Synergeia group performance suggests that the added value of Synergeia, namely the fact that it gives students the opportunity to reflect on their and their fellow students ideas, is a factor facilitating conceptual change.

## REFERENCES

- Gelman, R. (2000). The epigenesis of mathematical thinking. *Journal of applied developmental psychology*, 21(1), 27-37.
- Vamvakoussi, X. & Vosniadou, S. (in press). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*.
- Vosniadou, S. (1994) Capturing and modelling the process of conceptual change. In S. Vosniadou (Guest Editor) *Conceptual Change. Special issue of Learning and Instruction*, 4, 45-69.
- Vosniadou, S. (2001a). On the Nature of Naïve Physics. In M. Limon and L. Mason (Eds.). *Reframing the Processes of Conceptual Change*. Kluwer Academic Publishers.
- Vosniadou, S. (2001b). *How Children Learn.*, Educational Practices Series, 7, The International Academy of Education (IAE) and the International Bureau of Education (UNESCO).
- Vosniadou, S., (2002) Exploring the relationships between conceptual change and intentional learning. In G.M. Sinatra and P.R. Pintrich (Eds.), *Intentional Conceptual Change*. Mahwah, NJ: Lawrence Erlbaum Associates, 377-406.

